XXXVI. *Account of a new Inſtrument for meaſuring ſmall Angles, called the priſmatic Micrometer.* By the Rev. Nevil Maſkelyne, D. D. F. R. S. and Aſtronomer-Royal.

Read Dec. 18, 1777.

PRACTICAL aſtronomy was much benefited by the invention of the wire micrometer, for meaſuring differences of right aſcenſion and declination: nor did it receive leſs advantage from Mr. SAVERY's moſt ingenious invention of the divided object-glaſs micrometer, which has been rendered more commodious by the late Mr. JOHN DOLLOND's application of it to the object-end of a reflecting teleſcope, or the preſent Mr. PETER DOLLOND's application of it to the object-end of an achromatic refracting one.

But, valuable as the object-glaſs micrometer undoubtedly is, ſome difficulties have been found in the uſe of it, owing to the alterations in the focus of the eye, which are apt to cauſe it to give different meaſures of the ſame angle at different times. For inſtance, in meaſuring the Sun's diameter, the axes of the pencils of rays, which come through the two ſegments of the object-glaſs from

contrary limbs of the Sun, crossing one another at the focus of the telescope under an angle equal to that of the Sun's diameter, the union of the limbs of the two images of the Sun cannot appear perfect unless the eye be disposed to see objects distinctly which are placed at point of intersection. But if the eye be disposed to see objects distinctly, which are placed nearer the object-glass than the intersection is, the two limbs will appear separated by the interval of the axes of the pencils in that place; and if the eye be disposed to see objects distinctly, which are placed farther from the object-glass than the intersection is, the two limbs will appear to encroach upon each other by the distance of the axes of the pencils, after their crossing, taken at that place.

To explain this, let OV (plate XIX. fig. 1.) represent the centres of the two semi-circular glasses of the object-glass micrometer, separated to the distance OV from each other, subtending the angle O*a*V, equal to the Sun's diameter, at the point *a*, which is the common focus of the two pencils of rays having O*a* and V*a* for their axes, namely, those proceeding from contrary sides of the Sun, and passing through the contrary semi-circles; and let *d* be the eye-glass. It is evident, that if *d* be properly placed to give distinct vision of objects placed at the point *a*, the rays O*a*, V*a*, as well as all the other rays belonging

to thofe pencils, will be collected into one point upon the retina of the eye; and confequently, the two oppofite limbs of the two images of the Sun will feem to coincide, and the two images of the Sun to touch one another externally. But if the ftate of the eye fhould alter, the place of the eye-glafs remaining the fame, the eye will be no longer difpofed to fee the image formed at the point *a* diftinctly, but to fee an object placed at *ef*, nearer to or farther from the object-glafs diftinctly; and therefore an image will be formed on the retina exactly fimilar to the fomewhat confufed image formed by the rays on a plane perpendicular to their courfe at *ef*. Confequently, as the two cones of folar rays, *boa*, *cva*, formed by the two femi-circles, are feparated or encroach upon one another at this point of the axis by the diftance *ef*, the two images of the Sun will not feem to touch one another externally, but to feparate or to encroach upon one another by the interval *ef*. The error hereby introduced into the meafure of the Sun's diameter will be the angle *erf*, fubtended by *ef* at *r* the middle point between o and v, which is to *eaf* or o*a*v, the Sun's apparent diameter, as *ae* to *er*, or even to *ar*, on account of the fmallnefs of *ae* with refpect to *ar*.

Thefe confiderations concerning the caufe of a principal error that has been found in the object-glafs micro-

meter led me to inquire, whether ſome method might not be found of producing two diſtinct repreſentations of the Sun, or any other object, which ſhould have the axes of the pencils of rays, by which they are formed, diverging from one and the ſame point, or nearly ſo: and it occurred to me, that this might be done by the refraction of a priſm placed to receive part of the rays proceeding from the object, either before or after their refraction through the object-glaſs of a teleſcope. If the priſm be placed without the object-glaſs, the rays that are refracted through it will make an angle with the rays that paſs beſide it equal to the refraction of the priſm; and this angle will not be altered by the refraction of the object-glaſs afterwards. Conſequently, two images of an object will be repreſented, and the priſm ſo applied will enable us to meaſure the apparent diameter of any object, or any other angular diſtance which is equal to the refraction of the priſm. But if the priſm be placed within the object-glaſs, that is to ſay, between the object-glaſs and eye-glaſs, the angle meaſured by the inſtrument will vary according to the diſtance of the priſm from the focus of the object-glaſs, bearing the ſame ratio to the refraction of the priſm, as the diſtance of the priſm from the focus bears to the focal length of the object-glaſs.

Let

Let ACB (fig. 2.) represent the object-glass, and *d* the eye-glass of a telescope, and PR a prism placed to intercept part of the rays coming from an object, suppose the Sun, before they fall on the object-glass. The rays EE proceeding from the Eastern limb of the Sun, and refracted through the object-glass ACB without passing through the prism, will form the corresponding point of the Sun's image at *e*; and the rays WW proceeding in like manner from the Western limb of the Sun will be refracted to form the correspondent point of the Sun's image at *w*. But the rays 2E, 2E, 2W, 2W, proceeding in like manner from the Eastern and Western limbs of the Sun, and falling on the prism PR, and thence refracted to the object-glass ACB, will, after refraction through it, form the correspondent points of the Sun's image at 2*e*, 2*w*. Let the refraction of the prism be equal to the Sun's apparent diameter: in this case, at whatever distance the prism be placed beyond the object-glass, the two images of the Sun *we*, 2*w*2*e*, will touch one another externally at the point *e*2*w*; for the rays 2W, 2W, proceeding from the Western limb of the Sun being inclined to the rays EE proceeding from the Eastern limb in the angle of the Sun's apparent diameter, will, after suffering a refraction in passing through the prism equal to the Sun's apparent diameter, emerge

from the prifm and fall upon the object-glafs parallel to the rays EE, and confequently will have their focus 2*w* coincident with the focus *e* of the rays EE, and therefore the two images of the Sun *we*, 2*w*2*e*, will touch one another externally at the point *e*2*w*, and the inftrument will meafure the angle EC2W, and that only.

But if the prifm be placed within the telefcope, the angle meafured by the inftrument will be to the refraction of the prifm as the diftance of the prifm from the focus of the object-glafs is to the focal diftance of the object-glafs: or if two prifms be ufed to form the two images, with their refracting angles placed contrary ways, as reprefented in fig. 3. and 4. the angle meafured will be to the fum of the refractions of the prifms as the diftance of the prifms from the focus of the object-glafs is to the focal diftance of the object-glafs. For let ACB (fig. 3.) reprefent the object-glafs, and *d* the eye-glafs of a telefcope, and PR, RS, two prifms interpofed between them, with their refracting angles turned contrary ways, and the common fections of their refracting planes touching one another at R. The rays proceeding from an object, fuppofe the Sun, will be difpofed, by the refraction of the object-glafs, to form an image of the Sun at the focus; but part of them falling on one prifm, and part on the other, will be thereby refracted contrary ways, fo as to form two equal images *we*, 2*w*2*e*, which, if

if the refractions of the prisms be of proper quantities, will touch one another externally at the point $e2w$. Let ECN be the axis of the pencil of rays EE proceeding from the Sun's Eastern limb; and WCO the axis of the pencil of rays WW proceeding from the Sun's Western limb; and the point N the place where the image of the Sun's Eastern limb would be formed, and the point O where that of the Western limb would be formed, were not the rays diverted from their course by the refractions of the prisms. But by this means part of the rays EE, which were proceeding to N, falling on the prism PR, will be refracted to form an image of the Sun's Eastern limb at e, while others of the rays EE, which fall on the prism RS, will be refracted to form an image of the Sun's Eastern limb at $2e$. In like manner, part of the rays WW, which were proceeding to form an image of the Sun's Western limb at O, falling on the prism RS, will be refracted to form an image of the Sun's Western limb at $2w$ coincident with e, the point of the image correspondent to the Sun's Eastern limb; while others of the rays WW, which fall on the prism PR, will be refracted to form the image of the Sun's Western limb at w. The two images we, $2w2e$, are supposed to touch one another externally at the point $e2w$. The ray EFR, which belongs to the axis ECN, and is refracted by the prism PR to e, undergoes the refraction NRe, which (be-

cause

caufe fmall angles are proportional to their fines, and the fine of NR*e* is equal to the fine of its fupplement NRC) is to NCR as NC or C*e* is to NR or R*e*. In like manner, the ray WGR, which belongs to the axis WCO, and is refracted by the prifm RS to 2*w* or *e*, undergoes the refraction OR *e*, which is to OC*e* as OC or C*e* is to RO or R*e*; therefore, by compofition, ORN the fum of the refractions OR*e*, NR*e*, is to OCN the fum of the angles OC*e*, NC*e*, or the Sun's apparent diameter, as C*e* to R*e*; that is, as the focal diftance of the object-glafs to the diftance of the prifms from the focus of the object-glafs.

Or let the prifms PR, RS, be placed with their refracting angles P, S, turned from one another as in fig. 4.: the refraction of the prifm PR will transfer the image of the Sun from ON to *we*, and the refraction of the prifm RS will transfer the image ON to 2*w*2*e*, the two images 2*w*2*e*, *we*, touching one another externally at the point 2*ew*. Let ECN, WCO, be the axes of the pencils of rays proceeding from the two extreme limbs of the Sun, and N, O, the points where the images of the Sun's Eaftern and Weftern limbs would be formed by the object-glafs, were it not for the refraction of the prifms; the ray EFR, which belongs to the axis ECN, and is refracted by the prifm RS to 2*e*, undergoes the refraction NR2*e*; and the ray WGR, which belongs to the axis WCO, and is refracted by

by the prism PR to w, undergoes the refraction ORw. Now NC$2e$, part of the angle measured, is to NR$2e$, the refraction of the prism RS, as Rw to Cw; and OCw, the other part of the angle measured, is to ORw, the refraction of the prism PR, in the same ratio of Rw to Cw: therefore OCN, the whole angle measured, is to ORN, the sum of the refractions of the two prisms, as Rw to Cw; that is, as the distance of the prisms from the focus of the object-glass to the focal distance of the object-glass.

When the prisms are placed in the manner represented in fig. 3. the point e of the image we is illuminated only by the rays which fall on the object-glass between A and F, and the point $2w$ only by the rays which fall on the object-glass between B and G. Now the angles CRF, CRG, equal to the refractions of the prisms, being constant, the spaces FC, CG, will increase in proportion as the distances RF, RG, increase, and the spaces AF, GB, diminish as much; and therefore, the images at the point of mutual contact $e2w$ will be each illuminated by half the rays which fall on the object-glass when the prisms are placed close to the object-glass; but will be enlightened less and less the nearer the prisms are brought to the focus of the object-glass.

But

But when the prifms are placed in the manner fhewn in fig. 4. the images at the point of contact, as the prifms are removed from the object-glafs towards the eye-glafs, will be enlightened with more than half the rays that fall on the object-glafs, and will be moft enlightened when the prifms are brought to the focus itfelf; for the point *2e* of the image *2w2e* will be enlightened by all the rays EE that fall on the object-glafs between B and F, and the point *w* of the image *we* will be enlightened by all the rays WW which fall on the object-glafs between A and G. But the difference of the illuminations is not very confiderable in achromatic telefcopes, on account of the great aperture of the object-glafs; as the greateft fpace FG is to the focal diftance of the object-glafs, as the fum of the fines of the refractions of the prifms is to the radius.

There is a third way, and perhaps the beft, of placing the prifms, fo as to touch one another along their fides which are at right angles to the common fections of their refracting planes. In this difpofition of the prifms, the images will be equally enlightened, namely, each with half the rays which fall on the object-glafs, wherever the prifms be placed between the object-glafs and eye-glafs.

From

From what has been shewn it appears, that this instrument, which may be properly called the prismatic micrometer, will measure any angle that does not exceed the sum of the refractions of the prisms, excepting only very small angles, which cannot be taken with it on account of the vanishing of the pencils of rays at the juncture of the two prisms near the focus of the object-glass; that it will afford a very large scale, namely, the whole focal length of the object-glass for the greatest angle measured by it; and that it will never be out of adjustment; as the point of the scale where the measurement begins (or the point of o) answers to the focus of the object-glass, which is a fixed point for celestial objects, and a point very easily found for terrestrial objects. All that will be necessary to be done, in order to find the value of the scale of this micrometer, will be to measure accurately the distance of the prisms from the focus when the instrument is set to measure the apparent diameter of any object subtending a known angle at the centre of the object-glass, which may be easily found by experiment, as by measuring a base and the diameter of the object observed placed at the end of it, in the manner practiced with other micrometers: for the angle subtended by this object will be to the angle subtended by a celestial object, or very remote land object, when the

 distance

diſtance of the priſms from the principal focus is the ſame as it was found from the actual focus in the terreſtrial experiment, as the principal focal diſtance of the object-glaſs is to the actual focal diſtance in the ſaid experiment.

It will, I apprehend, be the beſt way in practice, inſtead of one priſm to uſe two priſms, refracting contrary ways, and ſo divide the refraction between them (as repreſented in fig. 3. and 4.). Achromatic priſms, each compoſed of two priſms of flint and crown-glaſs, placed with their refracting angles contrary ways, will undoubtedly be neceſſary for meaſuring angles with great preciſion by this inſtrument: and I can add with pleaſure, that I find by experiment made with this inſtrument, as it was executed by Mr. DOLLOND with achromatic priſms, ground with great care for this trial above a twelve-month ago, that the images after refraction through the priſms appear very diſtinct; and that obſervations of the apparent diameters of objects may be taken in the manner here propoſed with eaſe and preciſion.

Two or more ſets of priſms may be adapted to the ſame teleſcope, to be uſed each in their turn, for the more commodious meaſurement of different angles. Thus it may be very convenient to uſe one ſet of priſms for mea-

 ſuring

furing angles not exceeding 36′, and confequently fit for meafuring the diameters of the Sun and Moon, and the lucid parts and diftances of the cufps in their eclipfes; and another fet of prifms to meafure angles not much exceeding one minute, and confequently fit for meafuring the diameters of all the other planets. This latter fet of prifms will be the more convenient for meafuring fmall angles, on account of a fmall imperfection attending the ufe of this micrometer, as before mentioned; namely, that angles cannot be meafured with it when the prifms approach very near the focus of the object-glafs, the pencils of rays being there loft at the point where the prifms touch one another.

Upon the principles that have been here explained, a prifm placed within the telefcope of an aftronomical inftrument, adjufted by a plumb-line or level, to receive all the rays that pafs through the object-glafs, may conveniently ferve the purpofe of a micrometer, and fuperfede the ufe both of the vernier fcale and the external micrometer; and the inftrument may then be always fet to fome even divifion before the obfervation. Thus the ufe of a telefcopic level may be extended to meafure with great accuracy the horizontal refractions, the depreffion of the horizon of the fea, and fmall altitudes and depreffions of land objects. Time and experience

 will

will doubtlefs fuggeft many other ufeful applications of this inftrument.

A paper from the learned Abbé BOSCOVICH was read before this Society the ninth of laft June, defcribing a fimilar contrivance as an invention of the Abbé ROCHON, in which the Abbé BOSCOVICH himfelf alfo claims fome fhare; I therefore defire to acquaint this Society, that I communicated this invention to Mr. DOLLOND, and had it executed by him; and alfo fhewed the inftrument itfelf, fo executed, to my efteemed friend ALEXANDER AUBERT, efq. fellow of this Society, a gentleman very well qualified to judge of things of this nature, above a twelvemonth before the communication of the Abbé BOSCOVICH'S paper, as will appear from their written atteftations, drawn up at my defire, defcribing the particulars of the communication of this invention which I made to them fo long ago. May I be permitted to remark, that this inftrument having been executed by my directions, in feveral forms, by Mr. DOLLOND, between the months of March and Auguft, 1776, and fet up and tried at his houfe in the prefence of feveral of his workmen, could not be confidered as an abfolute fecret concealed from the public. However, I doubt not that the following atteftations of Mr. AUBERT and Mr. DOLLOND will fufficiently prove my title to this invention of the prifmatic micrometer;

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meter; and I take this opportunity of exhibiting to the Society the instrument itself, mentioned in Mr. DOLLOND's letter as executed by himself according to my directions, and sent to the Royal Observatory in the month of August 1776.

Greenwich,
December 11, 1777.

TO THE REV. DR. MASKELYNE.

St. Paul's Church-yard,
Nov. 22, 1777.

REV. SIR,

ACCORDING to your desire I send the following particulars of the experiments which were made by your directions, for compleating a new kind of micrometer for measuring small angles. About the beginning of April 1776, I received your first directions respecting this matter, which were to make two prismatic glasses or wedges of such angles that rays of light, which passed through them, should be refracted about 18′ of a degree: these were to be placed between the object-glass and eye-glass of an achromatic telescope about 30 inches long. The angular edges of the two prismatic glasses were to be placed in contact with each other; they were to be moved in a parallel position from the object-glass to the focus

focus of the eye-glafs, and to be of fuch a fize as to cover the aperture of the object-glafs when brought clofe to it. By the refraction of thefe wedges two images were formed in the telefcope, which were at the greateft diftance (about 36′) when the wedges were clofe to the object-glafs, and approached as they were moved towards its focus, where they united; fo that the whole focal diftance of the object-glafs was to be the length of the fcale for meafuring the angular diftance of the two images formed in the telefcope. When thefe wedges were applied, as above defcribed, the two images were found to be coloured to a great degree, occafioned by the refraction of the wedges. This defect you directed me to remove by making the prifmatic glaffes or wedges achromatic, on the fame principles as the achromatic object-glaffes; and, after fome difficulties, this was effected; the two images formed in the telefcope appeared free from colours and diftinct. The above experiments were made in a rough wooden tube, with an inconvenient method of moving the wedges by hand: in this ftate it was when fhewn to ALEXANDER AUBERT, efq. F. R. S. towards the end of May, 1776; after which you defired to have it done in a more compleat manner, in a brafs tube, with a means of turning the tube round to take angles in different directions, and a method of moving

moving the wedges with a screw. This was compleated about the middle of August in the same year, and then sent to the Royal Observatory. I have the honour to be,

REVEREND SIR,

Your obedient humble servant,

PETER DOLLOND.

I HEREBY certify, that in the month of May, 1776, the Rev. Mr. MASKELYNE, Astronomer-royal, produced to me, at Mr. DOLLOND'S house in St. Paul's church-yard, and in his presence, as a new invention of his own, an instrument for measuring small angles, consisting of two achromatic prisms or wedges applied between the object-glass and eye-glass of an achromatic telescope about 30 inches long, by moving of which wedges nearer to, or farther from, the object-glass, the two images of an object produced by them appeared to approach to, or recede from, each other, so that the focal length of the object-glass became a scale for measuring the angular distance of the two images.

London,
Nov. 27, 1777.

ALEX^R. AUBERT.

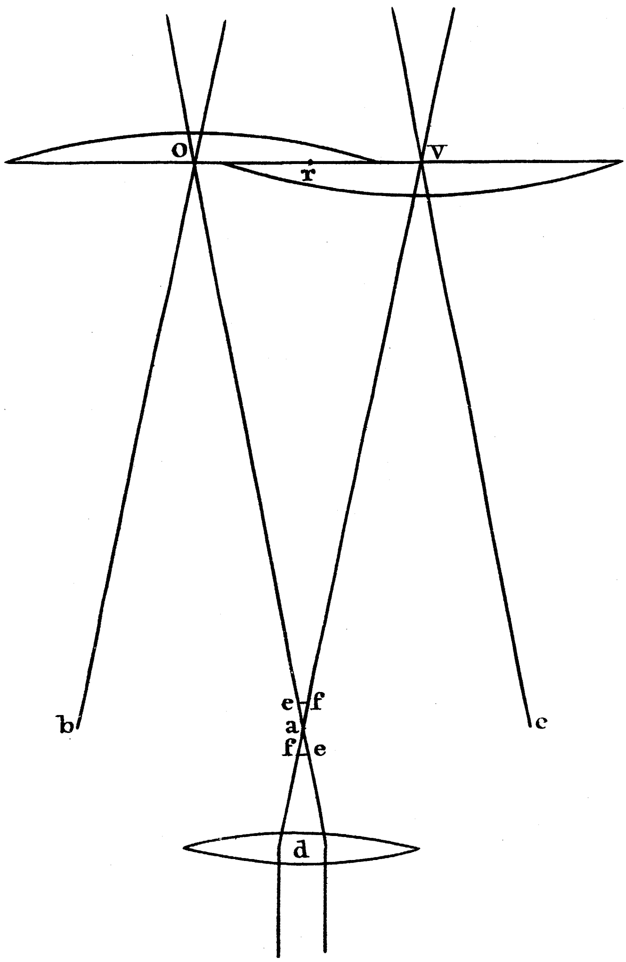


Fig: I.

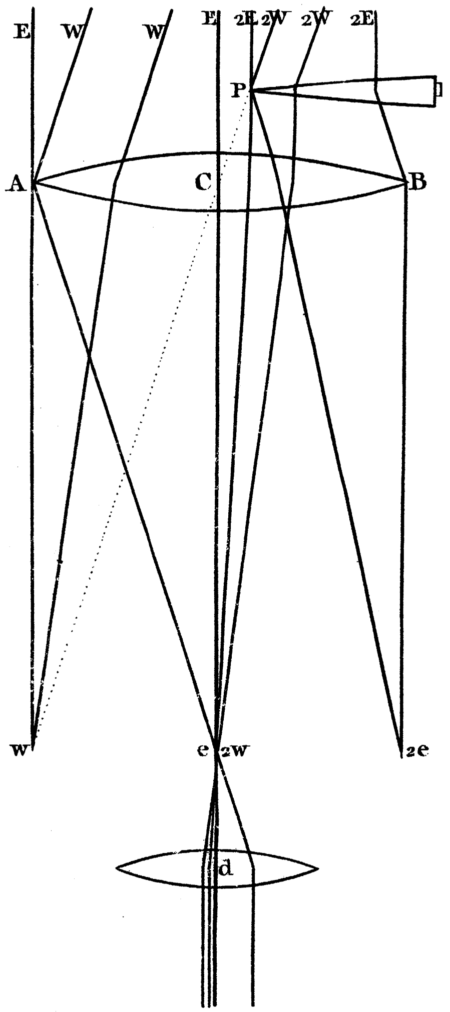
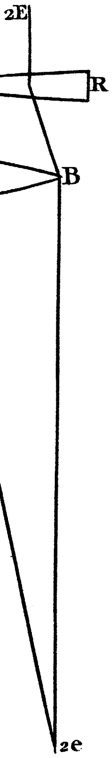


Fig: II.



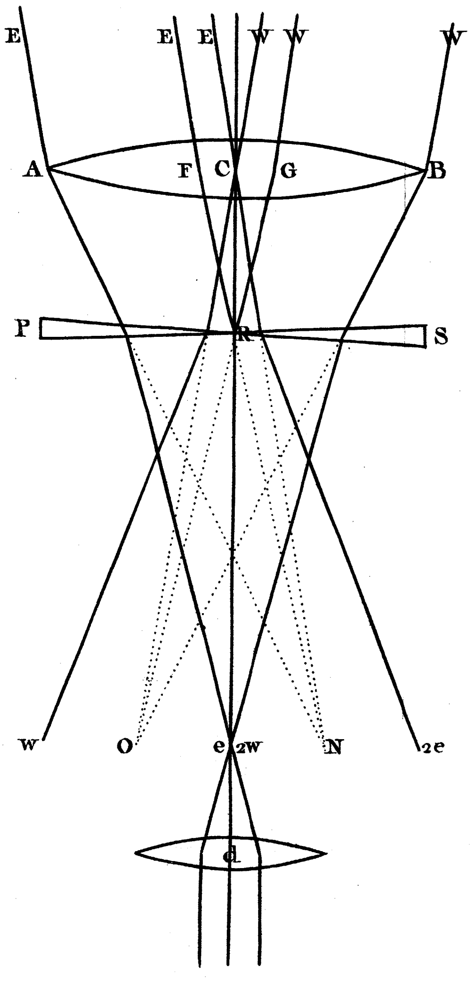


Fig: III.

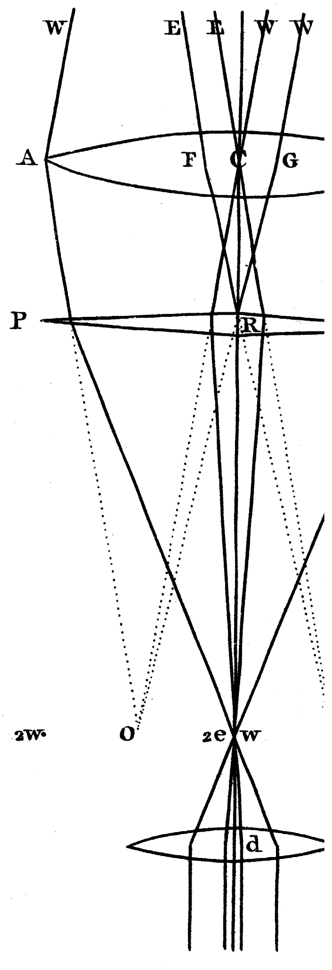
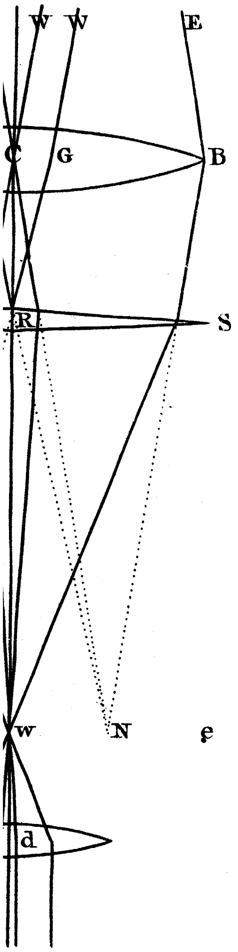


Fig: IV.



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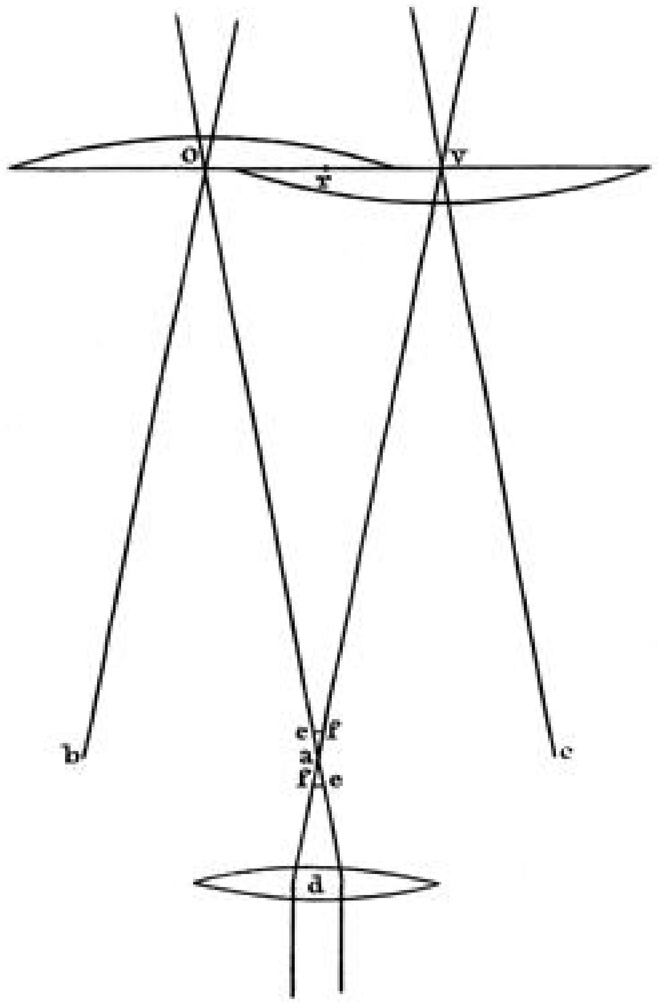

Fig: I.

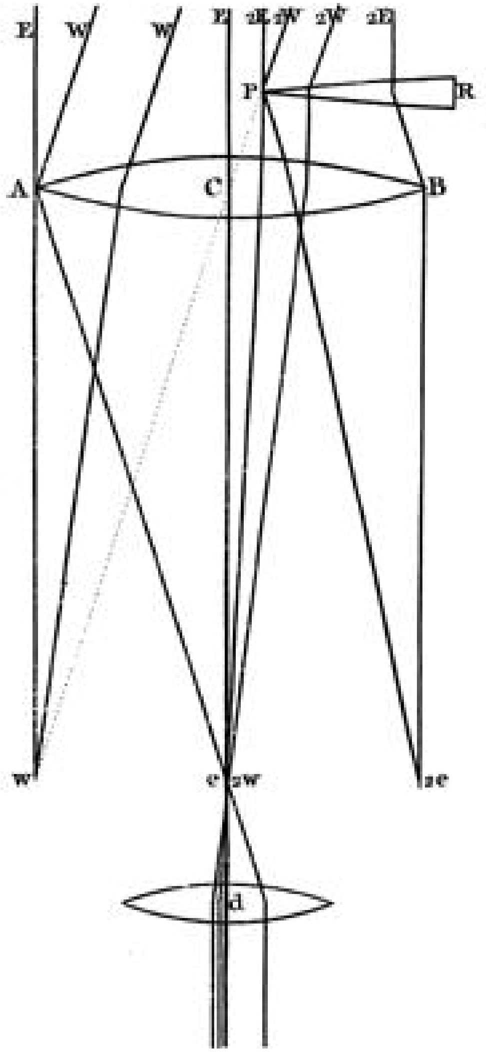

Fig: II.

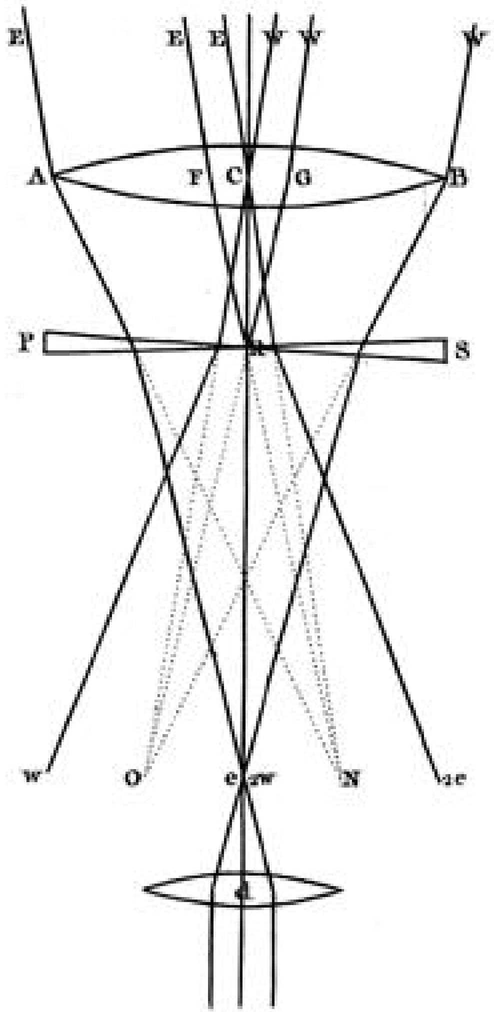

Fig: III.

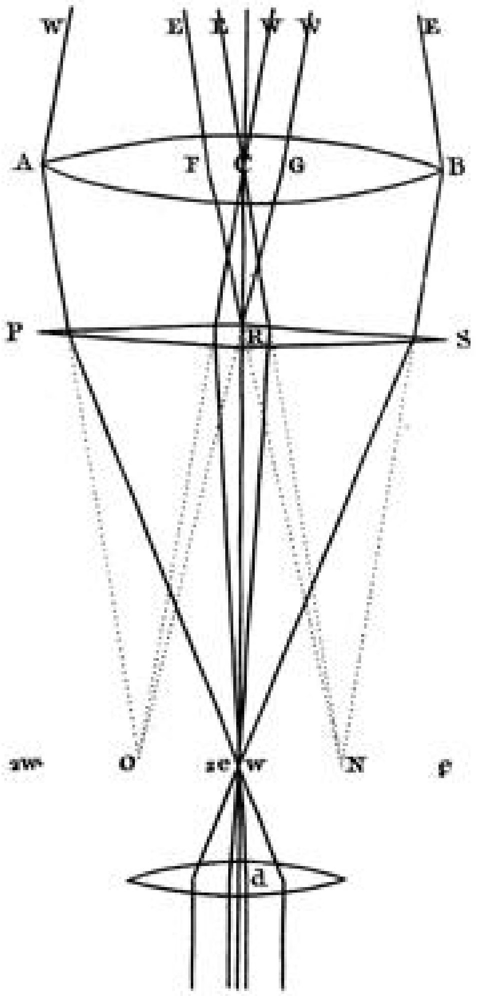

Fig: IV.